**INTERNATIONAL CONFERENCE**
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**IN SEVERAL COMPLEX VARIABLES III**
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## ABSTRACTS

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**Florian Bertrand**
American University of Beirut, Lebanon

### Metric properties of stationary discs

Stationary discs are special analytic discs introduced by Lempert as extremal discs for the Kobayashi metric for bounded strongly convex domains of $\mathbb{C}^n$. Afterwards, Poletsky showed that the stationarity condition for analytic discs is in fact the Euler-Lagrange equation corresponding to the Kobayashi extremal problem. Following Lempert's and Poletsky's works, I will discuss the description of extremal discs for higher order Kobayashi metrics. This is joint work with Giuseppe Della Sala and Jae-Cheon Joo.

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**Mehmet Celik**
Texas A&M University-Commerce, USA

### Obstructions for Compactness of Hankel Operators: Compactness Multipliers

We establish a connection between compactness of Hankel operators and geometry of the underlying domain through compactness multipliers for the $\overline{\partial}$-Neumann operator. In particular, we prove that any compactness multiplier induces a compact Hankel operator. We also generalize the notion of compactness multipliers to vector fields and matrices and then we use this generalization to generate compact Hankel operators. (*Joint with Yunus Zeytuncu, University of Michigan - Dearborn*)

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**Debraj Chakrabarti**
Central Michigan University, USA

### Function theory in annuli

Among the simplest examples of non-pseudoconvex domains are *annuli*, i.e., domains obtained by removing a relatively compact *hole* from a larger domain, the *envelope*. When the envelope and hole of an annulus are both smoothly bounded and pseudoconvex, Shaw (1985) proved $L^2$-estimates on the $\bar{\partial}$-problem, and obtained that the coholomologies are finite dimensional in appropriate degrees. Hörmander (2004) studied the annulus between two concentric balls, and determined the $L^2$-cohomology groups explicitely. We revisit this problem using a modified Morrey-Kohn-Hörmander formula adapted to annuli with holes which are only of class $C^{1,1}$. We also highlight a geometric obstruction that seems to play a key role in the problem.

This is joint work with Phil Harrington.

**Philippe Charpentier**
University of Bordeaux, France

### Weighted Bergman Projections and $\overline{\partial}$ equation in some pseudo-convex domains of finite type

In this talk I will present an overview of results obtained recently in collaboration with Y. Dupain and partially M. Mounkaila.

Let $\Omega$ be a smoothly bounded pseudo-convex domain in $\mathbb{C}^n$ of finite type. Let $\rho$ be a defining function of $\Omega$, $\eta$ a smooth strictly positive function and $r \geq 0$. We consider the weighted Bergman projection $P_\omega^\Omega$ of the Hilbert space $L^2(\Omega, \omega)$ for the weight $\omega = \eta(-\rho)^r$.

- For general $\Omega$, $P_\omega^\Omega$ satisfies $L^2_s(\Omega, \delta^r_{\partial\Omega})$ (directional) Sobolev estimates;
- When the rank of the Levi form of $\Omega$ is $\geq n-2$, for $r \in \mathbb{Q}_+$, $P_\omega^\Omega$ satisfies $L^p_s(\Omega, \delta^r_{\partial\Omega})$, $1 < p < +\infty$, and $\Lambda_\alpha(\Omega)$ estimates;
- When $\Omega$ is convex, if $\omega_0 = (-\rho_0)^r$, $r \in \mathbb{Q}_+$, $\rho_0$ being a special defining function of $\Omega$, $P_{\omega_0}^\Omega$ satisfies $L^p_s(\Omega, \delta^r_{\partial\Omega})$, $1 < p < +\infty$, and $\Lambda_\alpha(\Omega)$ estimates, and, for general weight $\omega$, $P_\omega^\Omega$ satisfies $L^p(\Omega, \omega)$ , $1 < p < +\infty$, and $\Lambda_\alpha(\Omega)$ estimates.

All the proofs are done proving first the regularity of $P_{\omega_0}^\Omega$ when $\omega_0$ is a special weight and extend it to $P_\omega^\Omega$ using estimates with gain for solutions of the $\overline{\partial}$-equation. In particular, for (lineally) convex domains new estimates are obtained.

**Paulo Cordaro**
University of Sao Paulo, Brazil

### Top degree cohomology in hypocomplex manifolds

In this talk we shall present some sheaf theoretical arguments, based on work by Serre, Andreotti-Grauert, Ramis-Ruget-Verdier, which can be applied to obtain results on global solvability in top degree for the differential complex associated to a hypocomplex structure. This is a joint work with M.R. Jahnke.

**Gian Maria Dall'Ara**
University of Vienna, Austria

ON EXPONENTIAL DECAY OF BERGMAN KERNELS ON COMPLETE HERMITIAN MANIFOLDS

I will report on recent joint work with Franz Berger and Son Duong concerning pointwise bounds for Bergman kernels associated to complete Hermitian manifolds endowed with a smooth positive measure. Our main result requires that the Ricci curvature is bounded from below and that the torsion of the Chern connection is bounded. We extend previous results of Christ, Delin and Ma and Marinescu among others.

**Giuseppe Della Sala**
American University of Beirut, Lebanon

A CHARACTERIZATION OF SPHERICITY BY ANALYTIC DISCS

Let $U$ be a strongly pseudoconvex domain of $\mathbb{C}^2$ with real-analytic boundary M. We prove the following sphericity result: suppose that all Segre varieties of $M$ intersect $U$ in a stationary disc. Then $M$ is locally biholomorphic to the sphere. This is joint work with F. Bertrand and B. Lamel.

**Makhlouf Derridj**
University of Normandy, Rouen, France

ON GEVREY VECTORS OF SECOND ORDER OF DIFFERENTIAL OPERATORS

Our aim is the study of the Gevrey regularity of the Gevrey vectors of a second order partial differential operator $P$, satisfying some Sobolev estimate. In particular , when $P$ is a Hörmander's operator of first kind , we obtain optimal Gevrey regularity for its Gevrey vectors.

**Luke Edholm**
University of Michigan at Ann Arbor, USA

IRREGULARITIES OF THE BERGMAN PROJECTION AND SUBSTITUTE OPERATORS

Let $\Omega \subset \mathbb{C}^n$ be a domain and define the Bergman space $A^p(\Omega)$ to be the space of holomorphic, $L^p$ functions on $\Omega$. Well established $L^p$ mapping properties of the Bergman projection known to hold on wide classes of domains are shown to fail for a certain kinds of bounded, pseudoconvex domains. This presents challenges to the development of holomorphic approximation theorems and to the classification of dual spaces. On these domains however, new substitute operators are constructed, avoiding the deficiencies present in the original Bergman projection. These operators allow for concrete statements of duality and approximation theorems in this setting.

This work is joint with Jeff McNeal and Debraj Chakrabarti.

**Siqi Fu**
Rutgers University-Camden, USA

### Spectral stability of the $\bar{\partial}$-Neumann Laplacian

The $\bar{\partial}$-Neumann Laplacian is an archetype of an elliptic operator with non-coercive boundary conditions. It is known that spectral behavior of the $\bar{\partial}$-Neumann Laplacian is intimately related to the underlying analytic and geometric structures. In this talk, we study spectral stability of the $\bar{\partial}$-Neumann Laplacian when either the operator or the underlying domain is perturbed. In particular, we establish spectral stability of the $\bar{\partial}$-Neumann Laplacian under the Kohn-Nirenberg elliptic regularization and under perturbation for smoothly bounded pseudoconvex domains. This talk is based on joint work with Weixia Zhu.

**Stefan Fürdös**
Masaryk University, Czech Republic

### Microlocal analysis in general ultradifferentiable classes

In this talk, we introduce an ultradifferentiable wavefront set with respect to a large family of ultradifferentiable classes and prove that it satisfies basic properties similar to those for the smooth wavefront set. The family of ultradifferentiable classes under investigation includes the Denjoy-Carleman (DC) classes defined by weight sequences and the Braun-Meise-Taylor (BMT) classes given by weight functions. The wavefront set presented in this talk generalizes in a natural way the one defined by Hörmander for DC classes and the one given by Albanese-Jornet-Oliaro for BMT classes, respectively.

This is joint work with David Nenning, Armin Rainer and Gerhard Schindl from the University of Vienna.

**Purvi Gupta**
Rutgers University, New Brunswick, USA

### Embeddings of compact real manifolds with prescribed polynomial hulls

We discuss some questions and recent results regarding the minimum embedding (complex) dimension of abstract compact (real) manifolds subject to constraints on their polynomial hulls. The primary challenge arises from the CR-singularities of a generic embedding. We will discuss why this is the case, and how they can be dealt with in the case when the set of CR-singularities is either of dimension zero or one. This is joint work with R. Shafikov.

**Gustavo Hoepfner**
University of Sao Carlos, Brazil

### On the Baouendi-Treves approximation theorem in Denjoy-Carleman classes

In this talk we shall discuss how to extend the seminal Baouendi-Treves approximation theorem to the classes of ultradifferentiable functions and distributions. This is a joint work with R. Medrado (UFC) and L. Ragognette (UFSCar).

**Xiaojun Huang**
Rutgers University, New Brunswick, USA

### Regular finite type conditions for pseudoconvex real hypersurfaces in $\mathbb{C}^n$ and the Bloom conjecture

This is joint work with Wanke Yin. We discuss the connection between the finite type condition defined by the Lie bracket of vector fields, the finite type condition in terms of the trace of the Levi form and the one by contact order with complex submanifolds.

**Bernhard Lamel**
University of Vienna, Austria

### Regularity of CR mappings in positive codimension

We discuss recent results, joint with N. Mir, on the regularity of CR mappings between smooth real submanifolds in complex spaces of different dimensions. Our main result relates existence of irregular mappings (in the sense that they fail to be smooth on an open set) on a minimal manifold to the existence of complex varieties tangent to infinite order to the target set, therefore providing optimal results for smooth regularity on a dense, open subset of the source.

**Bingyuan Liu**
University of Arkansas, USA

### Geometric analysis of the Diederich-Fornaess index

In this talk, we discuss the Diederich-Fornaess index in Several Complex Variables. A domain $\Omega \subset \mathbb{C}^n$ is said to be pseudoconvex if $-\log(-\delta(z))$ is plurisubharmonic in $\Omega$, where $\delta$ is a signed distance function of $\Omega$. The Diederich-Fornaess index has been introduced since 1977 as an index to refine the notion of pseudoconvexity. After a brief review of pseudoconvexity, we discuss this index form the point of view of geometric analysis. We will find an equivalent index associated to the boundary of domains and with it, we are able to obtain accurate values of the Diederich-Fornaess index for many types of domains.

**Andy Raich**
University of Arkansas, USA

Strong Closed Range Estimates: Necessary Conditions and Applications

While there has been a tremendous amount of effort in establishing sufficient conditions for closed range for the $\bar{\partial}$-operator, there have been no results in the past fifty years that work towards establishing necessary conditions for solvability. All of the techniques for proving closed range estimates establish a very version of the closed range estimate that Phil Harrington and I have dubbed Strong Closed Range Estimates. Using these Strong Closed Range Estimates as our hypothesis, we find geometric constraints reflected in conditions on the Levi form. I will discuss this work and, time permitting, other applications of SCRE, like solvability of the $\bar{\partial}$-Neumann problem in Sobolev spaces.

**Min Ru**
University of Houston, USA

Holomorphic curves into projective varieties intersecting general divisors

We establish a general Second Main Theorem type result (as well as Schmidt's subspace type theorem in Diophantine approximation) for holomorphic curves into the projective variety $X$ intersecting general divisor $D$, in terms of the (birational) Nevanlinna constant $\mathrm{Nevbir}(D)$. By computing $\mathrm{Nevbir}(D)$ using the filtrations, it recovers (almost all) previous known results in this direction, as well as derive some new results for divisors which are not necessarily linear equivalent on $X$. The notion $\mathrm{Nevbir}(D)$ is originally defined in terms of Weil functions for use in applications, and it is proved later that it can be defined in terms of local effectivity of Cartier divisors after taking a proper birational lifting. This is a joint work with Paul Vojta.

**Guokan Shao**
Academia Sinica, Taipei, Taiwan

$S^1$-equivariant index theorems and
Morse inequalities on complex manifolds with boundary

In this talk, we will present new versions of index theorems and Morse inequalities on complex manifolds with boundary. Let $M$ be a relatively compact open subset with connected smooth boundary $X$ in a complex manifold $M'$. Assume that $M$ admits a holomorphic $S^1$-action preserving the boundary $X$ and the $S^1$-action is transversal and CR on $X$. We claim that the $m$-th Fourier component of the $q$-th Dolbeault cohomology group $H^q_m(\overline{M})$ is of finite dimension. By using Poisson operator, we prove a reduction theorem which shows that the formulas about $H^q_m(\overline{M})$ in our main theorems involve only integrations over $X$. This talk is based on the joint work with Chin-Yu Hsiao, Rung-Tzung Huang and Xiaoshan Li.

**Laurent Stolovitch**
University of Nice, France

LINEARIZATION OF NEIGHBORHOODS OF EMBEDDINGS OF COMPLEX COMPACT MANIFOLDS

In this work, we address the following question due to Grauert: if a neighbourhood of a holomorphically embedded complex compact manifold is formally equivalent to another one, are they biholomorphically equivalent? We shall present the case where one of ambient manifold $M$ the compact manifold $C$ is embeded into is the normal bundle $N_C$ of $C$ into another $M^*$. This is joint work with X. Gong.

**Chloe Wawrzyniak -Urbanski**
Rutgers University, New Brunswick, USA

STABILITY OF HULLS OF THE $n$-SPHERE UNDER SMALL PERTURBATIONS

In following Bedford and Alexander, I will present a construction of the polynomial hull of a small perturbation of the standard $n$-sphere $S^n$ in $\mathbb{C}^n$ using attached analytic discs. This is one of the few results on stability of the polynomial hull when the set of singularities is non-empty and not isolated. We first utilize existing results from Kenig, Webster, and Huang about local hulls of holomorphy near nondegenerate elliptic CR singularities. We then solve a Riemann-Hilbert problem to construct attached discs away from the singularities. Finally, we show that the resulting smooth Levi-flat manifold, which is diffeomorphic to the hull of $S^n$, is the polynomially convex hull of the perturbed sphere. The work presented in this talk is joint work with Purvi Gupta.

**Dmitri Zaitsev**
Trinity College Dublin, Ireland

INVARIANT IDEAL AND SUBMODULE SHEAVES ON REAL HYPERSURFACES

Catlin's boundary systems is currently the only known approach to proving subelliptic estimates for general pseudoconvex finite type domains. We present some new geometric invariants aiming to better understand and simplify some of Catlin's technique.